

Search for $\frac{\Lambda^2}{p^2}$ corrections to the QCD running coupling

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We investigate the occurrence of power terms $\frac{\Lambda^2}{p^2}$ in the running QCD coupling by analysing non-perturbative measurements of $\alpha_s(p)$ at quite low momenta obtained from the lattice three-gluon vertex. Our study provides some evidence for such a contribution. The phenomenological implications of such a presence are reviewed.

1. Introduction and motivations

The Operator Product Expansion (OPE) is the standard way to parametrise non-perturbative QCD effects in terms of power corrections, the powers involved being uniquely fixed by the operator content of the theory. Anyway, due to the asymptotic nature of QCD perturbative expansions, i.e. due to renormalons ambiguities, power corrections are reshuffled between operators and coefficient functions [1]. It has recently been conjectured [2] that power corrections which are not *a priori* expected from OPE may appear in the expansion of physical observables, via much the same analysis as for renormalons, if one allows the presence of (UV-subleading) power corrections to $\alpha_s(p)$. The lattice community knows that perturbative logarithms are of course not the only contribution to the running coupling. Aim of the present work is to search for non-perturbative contributions to the running coupling in the form of power corrections, with the given power ($\frac{\Lambda^2}{p^2}$). Before briefly reviewing some theoretical arguments in support of this *theoretical prejudice*, a couple of considerations are in order. While power corrections to $\alpha_s(p)$ arise naturally in many physical schemes [3,4] and their occurrence cannot be excluded *a priori* in any scheme, their non-perturbative nature makes it very hard to as-

sess their scheme dependence, which is only very weakly constrained by the general properties of the theory.

1.1. A lesson from the Gluon Condensate

A stage on which all the considerations above apply is provided by the efforts towards a lattice determination of the Gluon Condensate, which is given in terms of Wilson loops W [2]. The OPE for W is to be written down as

$$W = W_0 + \frac{\Lambda^4 W_4}{Q^4} + \dots \quad (1)$$

W_0 being the contribution related to the identity operator and the second term being the “genuine” (dim=4) condensate (Q is the scale; on the lattice $Q \sim 1/a$). Perturbative contributions are present only in W_0 , so that a standard procedure to extract the condensate was to subtract from MonteCarlo measurements of Wilson loops their perturbative expansions. From general considerations the expected form of W_0 is

$$W_0 = \int_{\rho\Lambda^2}^{Q^2} \frac{dp^2}{p^2} \left(\frac{p^2}{Q^2}\right)^2 \alpha_s\left(\frac{p^2}{\Lambda^2}\right). \quad (2)$$

A renormalon analysis of this formula readily shows that a $O(\Lambda^4/Q^4)$ ambiguity is present in W_0 , thus preventing any unambiguous result from the above procedure. The situation is anyway even more intricate: actually, after having resummed W_0 (within the mentioned ambiguity),

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performing the subtraction leaves one with something that scales like $O(\Lambda^2/Q^2)$ [5]. The puzzle can be sorted out if one supposes in (2) contributions of the form $\alpha_s(p) \sim \frac{\Lambda^2}{p^2}$.

1.2. Static quarks potential and confinement

A nice argument to support the presence of $\frac{\Lambda^2}{p^2}$ contributions to $\alpha_s(p)$ comes from considerations involving confinement. Consider the interaction of two heavy quarks in the static limit [6]. Within the Born approximation one can obtain the static potential from

$$V(r) \propto \int d^3k \alpha_s(|\vec{k}|^2) \frac{\exp i\vec{k} \cdot \vec{r}}{|\vec{k}|^2}. \quad (3)$$

The above formula has been written down in the renormalon-style: plugging in a constant α_s yields the Coulomb potential ($V(r) \approx 1/r$), while the leading-logs expression for α_s generates various power corrections to the potential; however the string tension contribution $V(r) \sim Kr$ is missing. Such a contribution is obtained by plugging in a power correction to α_s of the form we are looking for. Notice that the above considerations are in a sense not new in the context of non-perturbative contributions to the running coupling [7]. Consider the “force” definition of the running coupling:

$$\alpha_{q\bar{q}}(Q) = \frac{3}{4} r^2 \frac{dV}{dr} \quad (Q = \frac{1}{r}), \quad (4)$$

where again $V(r)$ is the static interquark potential. By keeping into account the string tension contribution to $V(r)$, one obtains a $1/Q^2$ contribution, whose order of magnitude is given by the string tension itself. While this term was mainly considered as a sort of ambiguity, resulting in an indetermination in the value of $\alpha(Q)$ at a given scale, it can be interpreted as a clue for the existence of a $\frac{\Lambda^2}{p^2}$ contribution, providing also an estimate for the expected order of magnitude of it, at least in one (physically sound) scheme.

2. Fits to lattice data

The lattice data for $\alpha_s(p)$ that we used for our analysis were obtained by evaluating two-

and three-point off-shell Green’s functions of the gluon field in the Landau gauge and imposing non-perturbative renormalisation conditions on them, for different values of the external momenta [8]. Such a definition of the coupling corresponds to a momentum-subtraction renormalisation scheme in continuum QCD [9]. The data available at the moment are quite noisy and lattice artifacts are still to be fully assessed (they seem under control from the analysis in [8]). Given the particular ansatz to which we want to fit the data (perturbative expressions of given order plus a power correction), a peculiar momentum interval has to be singled out. On one hand, the momentum range should start well above the location of the perturbative Landau pole, but it should nonetheless include low scales where power corrections may still be sizeable. On the other hand, the requirement of keeping the effects of the finite lattice spacing under control in the numerical data for α_s induces a natural UV cutoff.

2.1. Two-loop analysis

At two-loop level we fit data to the formula

$$\alpha_s(p) = \frac{1}{b_0 L_2} - \frac{b_1 \log(L_2)}{b_0 (b_0 L_2)^2} + c_{2l} \frac{\Lambda_{2l}^2}{p^2} \quad (5)$$

where $L_2 = \log(p^2/\Lambda_{2l}^2)$. Notice that since the value of Λ is expected to carry a sizeable dependence on the order of the perturbative calculation, we append a subscript. We were able to check that out of power corrections of the form $(\frac{\Lambda_{2l}^2}{p^2})^z$ the value $z \approx 1$ is indeed the best choice, which thing we interpret as a confirmation of our theoretical prejudice. Our best fit singles out the values ($\Lambda_{2l} = 0.84(1), c_{2l} = 0.31(3)$), with a $\chi_{dof}^2 \leq 1.8$. The momentum interval which is best described is $p \sim 2 - 3$ GeV, fully consistent with the requirements we have already mentioned.

2.2. Three-loop analysis

Given the possible interplay between a description in terms of power corrections and our ignorance about higher orders, a three-loop analysis is compelling. A major obstacle for such an analysis is actually the fact that the first non-universal β -function coefficient b_2 is not known for our scheme. Therefore we also fitted data to

the formula

$$\alpha_s(p) = \frac{1}{b_0 L_3} - \frac{b_1 \log(L_3)}{b_0 (b_0 L_3)^2} + \frac{1}{(b_0 L_3)^3} \left(\frac{b_2^{eff}}{b_0} + \frac{b_1^2}{b_0^2} (\log^2(L_3) - \log(L_3) + 1) \right) + c_{3l} \frac{\Lambda_{3l}^2}{p^2}, \quad (6)$$

where $L_3 = \log(p^2/\Lambda_{3l}^2)$ and b_2^{eff} is an effective β -function coefficient to be determined from the fit. In order to gain insight, we started by putting $c_{3l} = 0$, obtaining the values ($\Lambda_{3l} = 0.72(1)$, $b_2^{eff} = 1.3(1)$), with $\chi_{dof}^2 \approx 1.8$, in a momentum range $p \sim 2 - 3$ GeV (dashed line in the figure). The value of b_2^{eff} is expected to be a reliable estimate of b_2 , as confirmed by simple arguments concerning the convergence properties of the expansion of our coupling in powers of other couplings. What is more important, the value of Λ_{3l} agrees with other determinations via perturbative matching [10]. By keeping into account also c_{3l} , moving around the same range for Λ_{3l} one obtains the values ($\Lambda_{3l} = 0.72(1)$, $b_2^{eff} = 1.0(1)$, $c_{3l} = 0.41(2)$), yielding a $\chi_{dof}^2 \approx 1.8$ in a momentum range $p \sim 1.8 - 3$ GeV (solid line in the figure). While b_2^{eff} still makes sense, it emerges that $c_{2l}\Lambda_{2l}^2 = 0.22(2) \text{ GeV}^2 \sim c_{3l}\Lambda_{3l}^2 = 0.21(2) \text{ GeV}^2$, which is just the order of the standard estimate for the string tension.

3. Conclusions and perspectives

Some preliminary evidence in support of a $\frac{\Lambda^2}{p^2}$ contribution to $\alpha_s(p)$ has been found, which appears at this stage disentangled from possible perturbative ambiguities: perturbative and non-perturbative (power) contributions do not mix in our formulae when upgrading from a two-loop to a three-loop description. While our results need to be further tested, still they provide evidence in support of the conjecture that “anomalous” (i.e., not accounted for by OPE) power corrections into current correlation functions and physical observables may be generated by power corrections to $\alpha_s(p)$.

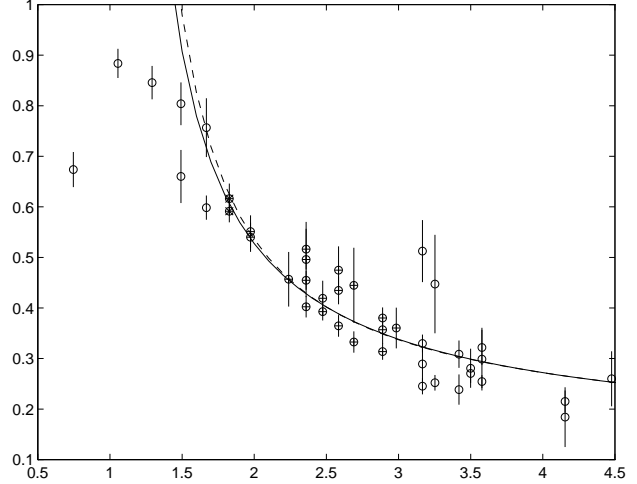


Figure 1. Three-loop fits to lattice data for the coupling, with and without power corrections.

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